**Solutions to Problem 1.**

a.
- For $0 \le t < 6$, $\Lambda(\tau) = \int_0^\tau 1\, da = \tau$
- For $6 \le t < 13$, $\Lambda(\tau) = \int_0^6 1\, da + \int_6^\tau 2\, da = 6 + (2\tau - 12) = 2\tau - 6$
- For $13 \le t < 24$, $\Lambda(\tau) = \int_0^6 1\, da + \int_6^{13} 2\, da + \int_{13}^\tau \frac{1}{2}\, da = 6 + 14 + \left(\frac{1}{2}\tau - \frac{13}{2}\right) = \frac{1}{2}(\tau + 27)$

b. $Z_8 - Z_2 \sim \text{Poisson}(\Lambda(8) - \Lambda(2)) = \text{Poisson}(8)$

$$\begin{aligned}\Pr\{Z_8 - Z_2 > 12\} &= 1 - \Pr\{Z_8 - Z_2 \le 12\} \\ &= 1 - \sum_{k=0}^{12} \frac{e^{-8}(8)^k}{k!} \approx 0.06 \\ E[Z_8 - Z_2] &= 8\end{aligned}$$

c. $Z_4 - Z_2 \sim \text{Poisson}(\Lambda(4) - \Lambda(2) = \text{Poisson}(2)$

$$\begin{aligned}\Pr\{Z_4 = 9 \,|\, Z_2 = 6\} &= \Pr\{Z_4 - Z_2 = 3 \,|\, Z_2 = 6\} \\ &= \Pr\{Z_4 - Z_2 = 3\} \\ &= \frac{e^{-2}(2)^3}{3!} \approx 0.18\end{aligned}$$

d. $Z_{1/4} \sim \text{Poisson}(\Lambda(1/4)) = \text{Poisson}(1/4)$

$$\begin{aligned}\Pr\{Z_{1/4} > 0\} &= 1 - \Pr\{Z_{1/4} = 0\} \\ &= 1 - \frac{e^{-1/4}(1/4)^0}{0!} \approx 0.22\end{aligned}$$

e. $Z_7 \sim \text{Poisson}(\Lambda(7)) = \text{Poisson}(8)$

$$\begin{aligned}\Pr\{Z_7 \ge 13\} &= 1 - \Pr\{Z_7 \le 12\} \\ &= 1 - \sum_{k=0}^{12} \frac{e^{-8}(8)^k}{k!} \approx 0.06\end{aligned}$$

**Solutions to Problem 2.**

a.
- For $0 \le t < 1$, $\Lambda(\tau) = \int_0^\tau 144\, da = 144\tau$
- For $1 \le t < 2$, $\Lambda(\tau) = \int_0^1 144\, da + \int_1^\tau 229\, da = 144 + 229(\tau - 1) = 229\tau - 85$
- For $2 \le t < 3$, $\Lambda(\tau) = \int_0^1 144\, da + \int_1^2 229\, da + \int_2^\tau 383\, da = 373 + 383(\tau - 2) = 383\tau - 393$
- For $3 \le t \le 4$, $\Lambda(\tau) = \int_0^1 144\, da + \int_1^2 229\, da + \int_2^3 383\, da + \int_3^\tau 96 = 756 + 96(\tau - 3) = 96\tau + 468$

b. $Z_{3.4} - Z_{1.75} \sim \text{Poisson}(\Lambda(3.4) - \Lambda(1.75)) = \text{Poisson}(478.65)$

$$E[Z_{3.4} - Z_{1.75}] = \Lambda(3.4) - \Lambda(1.75) = 478.65$$

c. $\Pr\{Z_{3.4} - Z_{1.75} > 700\} = 1 - \Pr\{Z_{3.4} - Z_{1.75} \le 700\}$

$$= 1 - \sum_{k=0}^{700} \frac{e^{-478.65}(478.65)^k}{k!} \approx 0$$

**Solutions to Problem 3.**

a. Note that $\Lambda(22) - \Lambda(18) = 87 - 74 = 13$, so $Z_{22} - Z_{18} \sim \text{Poisson}(13)$. Therefore,

$$\Pr\{Z_{22} - Z_{18} \le 12\} = \sum_{j=0}^{12} \frac{e^{-13}(13)^j}{j!} \approx 0.4631$$

b. Note that $\Lambda(24) - \Lambda(12) = 90 - 44 = 46$, so $Z_{24} - Z_{12} \sim \text{Poisson}(46)$. Therefore,

$$\begin{aligned}
\Pr\{Z_{24} \ge 80 \mid Z_{12} = 40\} &= \Pr\{Z_{24} - Z_{12} \ge 40 \mid Z_{12} = 40\} \\
&= \Pr\{Z_{24} - Z_{12} \ge 40\} \\
&= 1 - \Pr\{Z_{24} - Z_{12} \le 39\} \\
&1 - \sum_{j=0}^{39} \frac{e^{-46}(46)^j}{j!} \approx 0.8307
\end{aligned}$$

c. $\Lambda(24) = 90$ is the expected number of phone calls to the dispatch over the course of the entire day.